# Longitudinal diffusion by passing vehicles in road tunnels <br> Correlation equation on longitudinal diffusion coefficient 

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#### Abstract

A correlation equation for longitudinal diffusion coefficient of exhaust pollutants in road tunnels is proposed based on model and real-scale experiments. The equation correlates the diffusion coefficient with the mean equivalent resistance area of passing vehicles, traffic flow rate and vehicle Reynolds Number. An example of calculation is presented for tunnels with two-way traffic and the length limit of naturally ventilated tunnels is estimated.


## NOMENCLATURE

$A_{m} \quad=$ Equivalent resistance area of vehicle ( $=C_{D} A_{v}$ )
$A_{r} \quad=$ Cross-sectional area of tunnel
$A_{v} \quad=$ Frontal area of vehicle
$d_{t} \quad=$ Equivalent diameter of tunnel $\quad\left(=\left(4 A_{N} / \pi\right)^{1 / 2}\right)$
$\mathrm{m}^{2}$
m
$d_{v}=$ Equivalent diameter of vehicle $\quad\left(=\left(4 A_{v} / \pi\right)^{1 / 2}\right)$
$C=$ Concentration of pollutant
$C_{D} \quad=$ Aerodynamic drag coefficient of vehicle in tunnel traffic
$C_{D 0}=$ Aerodynamic drag coefficient of isolated vehicle in open air
D = Longitudinal diffusion coefficient in tunnel
$\mathrm{m}^{2} / \mathrm{s}$
G = Amount of pollutant that a vehicle emits per unit distance $\mathrm{m}^{3} /(\mathrm{m} \cdot \mathrm{veh})$
$L \quad=$ Length of tunnel including additional length ( $=L_{a i}+L_{r}+L_{a o}$ )
m
$L_{a i}, L_{a o}=$ Additional tunnel length at portals
m
$L_{r} \quad=$ Length of real tunnel
m
$l_{v}=$ Front to front spacing between vehicles on a lane m
$n \quad=$ Number of lanes
$N=$ Traffic flow rate
veh/s
$R_{e t}=$ Vehicle Reynolds Number ( $=v d_{v} / v$ )
$U_{r} \quad=$ Longitudinal through-flow velocity in tunnel
m/s
$v=$ Speed of vehicle $\mathrm{m} / \mathrm{s}$
$\alpha, \beta=$ Correction coefficient of drag for shadow and blockage effect
$\gamma_{L}=$ Large vehicle ratio
$\nu \quad=$ Kinematic viscosity
-
$\omega \quad=$ Emission of pollutant per unit length and time $(=G N)$
$\mathrm{m}^{2} / \mathrm{s}$

In road tunnels for automobiles, passing vehicles emit engine exhaust containing toxic gases and particulate matter (PM) into the surrounding air. Toxic gases such as CO and $\mathrm{No}_{\mathrm{x}}$ are hazardous for the health of passengers and PM such as diesel soot deteriorates driver visibility. It is therefore necessary to maintain the concentration of these pollutants in the tunnel below admissible limits stipulated for each constituent. Pollutants emitted in the tunnel must be removed by ventilation, the most common method of which is to transmit the polluted air longitudinally and blow it out from the tunnel portal as shown in Figure 1. This ventilation mechanism, in which pollutants are transmitted together with the air as a bulk, is called "convective mass transfer" or simply "convective ventilation" and is applied as a standard practice in most such ventilation. In typical text and reference books on ventilation, we can only find articles on ventilation theory based solely on convective pollutant transfer.

Studying a typical case of a naturally ventilated tunnel, with two-way (east- and west-bound) traffic, we find that there is a conventional concept among tunnel specialists that two-way traffic tunnels up to several hundred meters need no artificial ventilation. However, no practical explanation can be found regarding the limit of tunnel length. In two-way traffic tunnels there are two major reasons for longitudinal bulk movement of tunnel air and thus convective ventilation is induced by: 1) Unbalanced piston effect of east- and west-bound traffic (drag of vehicles apply longitudinal force to air column in the tunnel and push it like a moving piston), 2) East- or west-direction external natural wind force.

Such convective ventilation, however, relies on the traffic and natural wind condition. If we consider a situation where the drags of east- and west-bound vehicles completely cancel and there is no external wind, we may not expect any convective ventilation. For such cases we have had no theory to predict how high the pollutants concentration in the tunnel will be. As long as we rely only on convective ventilation, we have to consider the worst situation in which the air keeps stagnating, the consequence of which is a tunnel filled with raw emission gas. Such inconsistency, looking at it from experience, is caused because another mechanism of mass transfer, that is, diffusive mass transfer had been neglected.

The air in a road tunnel is continuously stirred and mixed by passing vehicles that are different in size and shape. The speed and front to front spacing of such vehicles varies as well. Therefore, the excited turbulence in the tunnel is highly space- and time-dependent, and is too complicated to handle. However, if we regard it as space- and time-averaged turbulence, we may be able to simply regard the tunnel space as a one-dimensional timeindependent turbulent field. Pollutant concentration $C$ becomes then the function of location,


Fig. 1 Ventilation in a road tunnel $C(x)$, as illustrated in Fig. 1. Turbulence has a strong mass transport effect and it transfers pollutants towards the low concentration side even when there is no bulk movement of air. The volume of a pollutant transmitted towards the x -direction, across a tunnel section per unit time, $P_{d}$, is proportional to the concentration gradient according to Fick's Law and can be expressed as:

$$
P_{d}=-A_{r} D d C(x) / d x
$$

where $A_{r}$ is the cross sectional area and $D$ the longitudinal diffusion coefficient.
Pollutants are also transmitted by convection across a tunnel section and this volume $P_{c}$ is obviously:

$$
P_{c}=A_{r} U_{r} C
$$

where $U_{r}$ is the average through-flow velocity toward the x -direction.
Conservation law of pollutants leads to the following concentration equation in the tunnel;

$$
\begin{equation*}
d\left(P_{c}+P_{d}\right) / d x=A_{r}\left(U_{r} d C / d x-D d^{2} C / d x^{2}\right)=\omega \tag{1}
\end{equation*}
$$

where $\omega$ denotes the volume of a pollutant emitted in unit length of a tunnel per unit time. The second term of the above equation expresses the diffusive pollutant transfer and restricts the increase of pollutant concentration, even when there is no through-flow in the tunnel, $U_{r}=0$.

The present paper proposes a practical method for estimating longitudinal diffusion coefficients (LDCs). The proposed correlation equation for LDCs has been worked out using the result of a series of experiments that have been conducted in model and real tunnels with one- and two-way traffic.

The first investigation on LDCs was done in a model circular tunnel in which simulated bodies were driven at various speeds in two-way traffic. Effects by the vehicle Reynolds Number and vehicle spacing on LDCs revealed by this investigation were reported (Ref.1). LDCs were further studied in a model tunnel with one-way traffic (Ref.2). After model experiments revealed the basic characteristics of LDCs, experiments were conducted in actual tunnels using a tracer method (Ref.3,4). A given amount of tracer gas ( $\mathrm{SF}_{6}$ ) was released instantly into the tunnel (by bursting a gas-filled balloon) and the variation of concentration was measured at a location about 1 km downstream the tracer source. LDCs can be obtained by applying Fick's Law to actually measured concentration distribution results(refer to the appendix).LDC data obtained on actual phenomena furthered our understanding on the phenomena and made it possible to estimate effects by tunnel diameter, as well as by the number of lanes and type of traffic (one- or two-way).

The present paper proposes a highly generalized correlation equation for LDCs, applicable for various tunnel geometry, traffic and emission conditions. As an example of application, the limiting length of a naturally ventilated, two-way traffic tunnel is determined under the condition that the pollutant concentration in the tunnel does not exceed the allowable limit.

## 2 CORRELATION EQUATION FOR LONGITUDINAL DIFFUSION COEFFICIENT

### 2.1 LDC and dimensional analysis

Main independent parameters that determine flow and mixing phenomena in a tunnel are:
(1) Parameters related with vehicles:
equivalent diameter of vehicle $d_{v}$, drag coefficient of vehicle in tunnel $C_{D}$ (under blockage and shadow effect), speed of vehicle $v$, front to front spacing between vehicles $l_{v}$ and
(2) Parameters related with the tunnel:
equivalent diameter of tunnel $d_{t}$, kinematic viscosity of air $v$.

LDC $D$ is the index to indicate the strength of turbulent mixing, and therefore can be expressed as a function of all these parameters;

$$
\begin{equation*}
D=f_{l}\left(d_{t}, d_{v}, v, C_{D}, v, l_{v}\right) \tag{2}
\end{equation*}
$$

The above can be transformed into an equation for non-dimensional quantities based on dimensional analysis;

$$
\begin{equation*}
D /\left(v d_{v}\right)=f_{2}\left(v d_{v} / v, d_{v} / d_{t}, C_{D}, l_{v} / d_{v}\right) \tag{3}
\end{equation*}
$$

This means that non-dimensional LDC, $D /\left(v d_{v}\right)$, is a function of four non-dimensional parameters, i.e. vehicle Reynolds Number $R_{e f}=v d_{v} / v$, diameter ratio of vehicle to tunnel $d_{v} / d_{t}$, drag coefficient of vehicle $C_{D}$ and vehicle spacing to diameter ratio $l_{v} / d_{v}$.

### 2.2 Experimental results and their interpretations

All LDC data known so far are plotted in Figure 2 using nondimensional LDC $D /\left(\begin{array}{ll}v & d_{v}\end{array}\right)$ shown along the vertical axis and vehicle Reynolds Number $R_{e t}$ shown along the abscissa in logarithmic scale. The data include all measured LDC in model and real tunnels. When we observe the plotted data closely, we may draw lines of constant vehicle spacing to diameter ratio $l_{v} / d_{v}$ for each data group of model and real tunnels as shown in the figure. It is clear that all these constant $l_{v} / d_{v}$ lines have the same inclination and this leads to the very important conclusion that non-dimensional LDC is proportional to the power 0.13 of


Fig. 2 Relationship between passing vehicle Reynolds number and non-dimensional diffusion coefficient vehicle Reynolds Number, provided other parameters stay constant.

Considering $f_{2}$ is proportional to $R_{e t}{ }^{0.13}$, Eq.(3) can be rewritten;
$D /\left(v d_{v}\right)=f_{3}\left(d_{v} / d_{t}, C_{D}, l_{v} / d_{v}\right) R_{e t}{ }^{0.13}$
where $f_{3}$ decreases with the increase in spacing to diameter ratio $l_{v} / d_{v}$ and is approximately inversely proportional.

### 2.3 Forming of the correlation equation

For the convenience of practical application, we hereinafter consider the correlation equation for LDC with dimensions. In the analysis of traffic data, traffic flow rate $N$, number of vehicles crossing a tunnel section per unit time, are commonly used to indicate the traffic intensity. Traffic flow rate per lane is obviously equal to $v / l_{v}$ if we consider the mean value of vehicle spacing, and the total traffic flow $N$ of a tunnel with $n$ lanes becomes:

$$
N=n v / l_{v} \text { or } v=N l_{v} / n \text { and } v d_{v}=N l_{v} d_{v} / n=N d_{v}^{2}\left(l_{v} / d_{v}\right) / n
$$

Introducing the above relation to Eq.(4), LDC can be expressed as:

$$
\begin{align*}
& D=\left[f_{3}\left(d_{v} / d_{t}, C_{D}, l_{v} / d_{v}\right)\left(l_{v} / d_{v}\right) / n\right] N d_{v}^{2} R_{e t}^{0.13} \\
& =f_{4}\left(d_{v} / d_{t}, C_{D}, l_{v} / d_{v,} n\right) N d_{v}^{2} R_{e t}^{0.13} \tag{5}
\end{align*}
$$

Since $f_{3}$ is nearly inversely proportional to $l_{v} / d_{v}, f_{3}$ times $l_{v} / d_{v}$ in [] of the above equation would loose most of its dependence on $l_{v} / d_{v}$. The function in [ ] is replaced by a new influence function $f_{4}$ that represents the dependence on four parameters, $d_{v} / d_{t}, C_{D}, l_{v} / d_{v}$ and $n$.

In the following discussion, Eq.(5) is further converted with the intention that the influence function has the least dependence on parameters. The only physical quantity with dimensions in Eq.(5) is $N d_{v}{ }^{2}$ in $\mathrm{m}^{2} / \mathrm{s}$. We may substitute the quantity by another with the same dimensions so as to make function $f_{4}$ as simple and comprehensive as possible. After repeated trial and correction, we reached a simple chain of understanding that an LDC is affected solely by turbulence intensity in the tunnel; i.e. turbulence is maintained by continuous disturbance of passing vehicles; turbulence energy or intensity is proportional to the aerodynamic input from vehicles; aerodynamic input of a vehicle is equal to aerodynamic drag multiplied by vehicle speed; turbulence intensity is thus correlated to the total aerodynamic input from passing vehicles.

The aerodynamic drag of a vehicle is proportional to $C_{D} A_{v} v^{2}$ where $A_{v}$ is the frontal area of the vehicle. It must be noted that drag coefficient $C_{D}$ is the value in an actual tunnel traffic condition and is different from the catalogue value $C_{D O}$ that is measured in open air without interaction from the foregoing vehicle. If we consider mean values in the following, the aerodynamic input per unit length of tunnel per unit time is proportional to $C_{D} A_{v} v^{2} N$. This quantity is the direct source of turbulence energy and must be determinant to LDC. Since the influence of vehicle speed $v$ has been taken into account as the contribution of vehicle Reynolds Number, the remaining quantity $C_{D} A_{v} N$ is finally introduced to replace $N d_{v}{ }^{2}$ in Eq.(5).

$$
\begin{align*}
D= & f_{5}\left(d_{v} / d_{t}, C_{D}, l_{v} / d_{v} n\right) C_{D} A_{v} N R_{e t}{ }^{0.13} \\
& =f_{5} A_{m} N R_{e t}^{0.13} \tag{6}
\end{align*}
$$

where $A_{m}=C_{D} A_{v}$ is the equivalent resistance area of vehicle and denotes the frontal area when $C_{D}$ is assumed to be unity.

### 2.4 Estimation of drag coefficient in a tunnel

It is not difficult to know the drag coefficients of vehicles, since they are given either in the catalogue of individual makes or information otherwise provided. They are, however, values of an isolated vehicle in open air. Drag coefficient $C_{D}$ of a running vehicle in a tunnel deviates from the catalogue value $C_{D O}$ because of two reasons: (1) Shadow effect of foregoing vehicle that decreases drag. (2) Blockage effect by tunnel wall that increases drag.

$$
\begin{equation*}
C_{D}=\alpha \beta C_{D 0} \tag{7}
\end{equation*}
$$

where $\alpha$ and $\beta$ denote correction factors for shadow and blockage effect, respectively.
Information on shadow effect was first given by Haerter (Ref.7), who showed that drag starts to attenuate when vehicle spacing to diameter ratio $l_{v} / d_{v}$ becomes smaller than 16.75 . The experimental result is expressed by the following formula:

$$
\begin{array}{ll}
\text { For } l_{\nu} / d_{v} \geqq 16.75 & \alpha=1 \\
\text { For } l_{\nu} / d_{v}<16.75 & \alpha=-2.3510^{-3}\left(l_{v} / d_{v}\right)^{2}+9.906410^{-2}\left(l_{v} / d_{v}\right) \tag{8}
\end{array}
$$

In an enclosed space like a tunnel, the external flow around a vehicle is accelerated faster than that in an open space, resulting in an increase of drag due to the blockage effect. This effect is the function of the vehicle to the tunnel area ratio $A_{v} / A_{r}$ and will diminish when the ratio is sufficiently small. Minato (Ref.8) conducted series of model experiments and expressed the correlation factor $\beta$ by the following simple empirical formula:

$$
\begin{equation*}
\text { For } A_{v} / A_{r}<0.25 \quad \beta=1+3.4\left(A_{\vee} / A_{r}\right) \tag{9}
\end{equation*}
$$

There is such a variety of vehicles in real traffic, ranging from small passenger cars to large freight trucks. It is practical to classify all vehicles into two groups for statistical reasons, namely small and large vehicles. The boundary in Japanese practice is given below (Note). By introducing ratio of large vehicles in total, $\gamma_{L}$, the mean equivalent resistance area $A_{m}$ in a tunnel is determined as the weighed mean of two groups:

$$
\begin{align*}
A_{m} & =\left(C_{D} A_{v}\right)_{L} \gamma_{L}+\left(C_{D} A_{v}\right)_{S}\left(1-\gamma_{L}\right) \\
& =\alpha\left[\left(\beta C_{D 0} A_{v}\right)_{L} \gamma_{L}+\left(\beta C_{D 0} A_{v}\right)_{S}\left(1-\gamma_{L}\right)\right] \tag{10}
\end{align*}
$$

As seen in Eq. (10) we applied a common correction factor $\alpha$ for two groups from the viewpoint of practical usage. The vehicle spacing $l_{v}$ in Eq. (8) is calculated as an averaged value of all lanes, while the vehicle diameter $d_{v}$ is calculated by the weighed mean of two groups; $d_{v}=d_{v L} \gamma_{L}+d_{v S}\left(1-\gamma_{L}\right)$.

From statistical data on vehicles running on Japanese highways, we obtained the following mean values that are used in our LDC analysis.

$$
\begin{aligned}
& \text { Small vehicles: } C_{D O}=0.32, A_{v}=2.3 \mathrm{~m}^{2} \\
& \text { Large vehicles: } C_{D O}=0.63, A_{v}=7.2 \mathrm{~m}^{2}
\end{aligned}
$$

Using the above local values, the most convenient equation for the equivalent resistance area $A_{m}$ of a tunnel can be expressed as:

$$
\begin{equation*}
A_{m}=\alpha\left[0.74+5.8 / A_{r}+\left(3.8+105 / A_{r}\right) \gamma_{L}\right] \tag{11}
\end{equation*}
$$

where $A_{m}$ and $A_{r}$ are given in metric unit, $\mathrm{m}^{2}$.

[^0]
### 2.5 Correlation equation for LDC

After we calculated the equivalent resistance area $A_{m}$ of all tested traffic conditions, LDC data were re-plotted in Figure 3 as the relation of $D$ to $A_{m} N R_{e t}{ }^{0.13}$ in logarithmic scale. As seen from the figure, all plotted data are located on a straight line of unit inclination with an amazingly high correlation coefficient of 0.99 . The result shows that influence function $f_{5}$ of Eq.(6) is least dependent of any parameters and keeps a constant value of 10.5 . The final correlation equation for LDC is expressed as:

$$
\begin{equation*}
D=10.5 A_{m} N R_{e t}{ }^{0.13} \tag{12}
\end{equation*}
$$

As seen from Figure 3, all experimental data fall within $\pm 24 \%$ of the correlation value.


Fig. 3 Relationship between diffusion coefficient $D$ and $A_{m} N R_{e t}{ }^{0.13}$
Uncertainties(Ref.9): $A_{m} N R_{e t}{ }^{0.13}: \pm 14 \%, D: \pm 24 \%$

The features of the above correlation equation are:

1. A very wide range of vehicle Reynolds Number $R_{e f}, 10^{3}$ to $10^{7}$ is covered and is applicable to both model and actual tunnels.
2. It is applicable to both one- and two-way traffic.
3. The correlation has been proven to work for two- and three-lane tunnels.
4. As for traffic data, only the traffic flow rate $N$, large vehicle ratio $\gamma_{L}$ and vehicle speed $\boldsymbol{v}$ are required as input factors for the correlation.

The simplicity and comprehensiveness of the obtained correlation is beyond our expectation and the success could be attributed to the sound physical interpretation of the phenomena that lead to the forming of Eq.(6).

## 3. APPLICATION OF LONGITUDINAL DIFFUSION COEFFICIENT

### 3.1 Boundary conditions

As an example of the application of LDC, concentration distribution of pollutants in naturally ventilated road tunnels is discussed considering convective and diffusive mass transfer. The concentration is obtained by integrating Eq.(1) with properly assumed boundary conditions.

Let's consider a tunnel without any artificial ventilation equipment as shown in Fig. 4. The through-flow velocity $U_{r}$ in the tunnel is so determined that total drags of passing vehicles balance with the wall friction of the through-flow. We consider $U_{r}$ as a constant that can be estimated by a commonly known method.

For the integration of Eq.(1), setting a proper boundary conditions is rather difficult. The tunnel shown in Fig. 4 blows out polluted air from the right portal and this air jet begins to mix with ambient fresh air. The pollutant concentration on the road decreases gradually as indicated by one-dotted dash line. This attenuation is the result of three-dimensional mixing of the jet in open air and is obviously different from the situation in the tunnel.


Fig. 4 Schematic diagram of additional tunnel length at portals
The concentration at an actual portal is not zero but holds a finite value as shown in the figure. Fig. 4 suggests a concept of adding a virtual tunnel to either side of the tunnel. We add a tunnel, length $L_{a i}$ (intake side) and $L_{a o}$ (outlet side), to left and right side of portals and assume that the concentration is zero at the portals of the added tunnels. If the assumed length of added
tunnels is appropriate, the concentration in the actual tunnel should match. The total length of the tunnel $L$ is the sum of real length $L_{r}$ and added virtual ones:

$$
\begin{equation*}
L=L_{a i}+L_{r}+L_{a o} \tag{13}
\end{equation*}
$$

As for the length of additional tunnel we have following estimated factors:

1. If a cross wind (external wind blowing perpendicular to the road) is sufficiently strong, air near the portals is replaced with fresh air instantly and additional length can be neglected.
2. If there is a substantial through-flow $U_{r}$ in the tunnel, additional length of intake side $L_{a i}$ can be neglected.
3. If there is a moderate cross wind that constitutes $60 \%$ of through-flow $U_{r}$, additional length of outlet side is about three-fold the tunnel diameter, $L_{a o} \cong 3 d_{t}$. (Ref.10)
4. If there is no through-flow and no cross wind, additional length is about three-fold of the tunnel diameter on both sides, $L_{a i}=L_{a o} \cong 3 d_{t}$. (Ref.1)

We expect further findings on additional length by future experiments.

### 3.2 Concentration distribution

Eq.(1) is integrated with the boundary conditions $C=0$ at $x=0$ and $L$, and the result is expressed as:

$$
\begin{equation*}
C(x)=\frac{\omega}{A_{r} U_{r}}\left[x-L \frac{\exp \left(U_{r} x / D\right)-1}{\exp \left(U_{r} L / D\right)-1}\right]=C_{0} \frac{8}{k}\left[\frac{x}{L}-\frac{\exp (k x / L)-1}{\exp (k)-1}\right] \tag{14}
\end{equation*}
$$

where reference concentration $C_{0}=\omega L^{2} / 8 D A_{r}$ and convective to diffusive transfer ratio $k=U_{r} L / D$.

The above solution cannot be applied when there is no through-flow, $U_{r}=0$. The concentration distribution in this case can be obtained by neglecting the convective term and integrating the remaining diffusive term of Eq.(1) and thus:

$$
\begin{equation*}
C(x)=\frac{\omega}{2 D A_{r}}(L-x) x=4 C_{0}[1-(x / L))(x / L) \tag{15}
\end{equation*}
$$

Concentration distribution is calculated by the above equations and the result is shown in Fig. 5 , where normalized concentration $C / C_{0}$ in the tunnel is plotted for various $k$ values. It is clear that the concentration is highest when there is no through-flow, $U_{r}=k=0$ and it becomes lower as $U_{r}$ or $k$ increases.

Maximum concentration $C_{\text {max }}$ occurs at the tunnel center when $U_{r}$ is zero and coincides with the reference concentration $C_{0}$. At a finite through-flow $U_{r}$, the maximum concentration $C_{\max }$ and the location $x_{m}$ are expressed as:

$$
\begin{align*}
& x_{m}=(L / k) \ln [(\exp (k)-1) / k]  \tag{16}\\
& \left.C_{\text {max }}=\left(8 C_{0} / k^{2}\right) \ln [(\exp (k)-1) / k]-(\exp (k)-1-k) /(\exp (k)-1)\right\} \tag{17}
\end{align*}
$$

As can be gathered from Fig. 5, the location of maximum concentration shifts from the tunnel center toward the outlet portal along the increase in through-flow.


Fig. 5 Concentration distribution
$\left[L=2051.6 \mathrm{~m}, A_{r}=58 \mathrm{~m}^{2}, d_{i}=8.6 \mathrm{~m}, N=0.556 \mathrm{veh} / \mathrm{s}\right.$,
$\left.v=16.67 \mathrm{~m} / \mathrm{s}, \quad \gamma_{L}=20 \%, D=76.4 \mathrm{~m}^{2} / \mathrm{s}, d_{\nu}=1.975 \mathrm{~m}\right]$

### 3.3 Limiting length of naturally ventilated two-way tunnels

The concentration becomes highest when there is no through-flow, and this situation happens when piston actions of opposing traffic in a two-way tunnel cancel each other. The maximum concentration matches in this case with the reference concentration $C_{0}=\omega L^{2} / 8 D A_{r} \quad$ After replacing $C_{0}$ by $C_{\max }, \omega$ by $G N$ and $D$ by Eq.(12), the limiting length of the real tunnel, $L_{\text {lim }}$, can be determined by:

$$
\begin{equation*}
L_{\text {lim }}=\left(8 C_{\max } D A_{r} / \omega\right)^{1 / 2}-\left(L_{a i}+L_{a o}\right)=\left(84 C_{\max } A_{m} R_{e t}^{0.13} A_{r} / G\right)^{1 / 2}-\left(L_{a i}+L_{a o}\right) \tag{18}
\end{equation*}
$$

where $G$ is the amount of pollutant that a vehicle emits in a unit running distance, $\mathrm{m}^{3} / \mathrm{m}$ veh.
The numerical result is shown in Fig. 6 (a) and (b) for two different pollutants, that is, $\mathrm{NO}_{\mathrm{x}}$ and PM. All parameters necessary for the calculation are listed in Table 1. Emissions of vehicles are estimated from the statistical data measured on Japanese highways (Ref.11) shown in Fig. 7 (a) and (b), and the best fitted curves for large and small vehicle groups are used in the calculation.


Fig. 6 Limiting length of naturally ventilated two-way tunnels $\left(\boldsymbol{U}_{\boldsymbol{r}}=0\right)$


Fig. 7 Statistical data on $\mathbf{N O}_{x}$ and PM emission on Japanese highways
Table 1 Parameters used for calculation of limiting length

| Parameters | Values |  |
| :---: | :---: | :---: |
| $A_{r}$ | $58 \quad(\mathrm{~m}$ |  |
| $d_{t}$ | 8.6 (m) |  |
| $A_{m}$ | Eq.(11) (m) |  |
| $C_{\text {max }}$ for NOx | 15 (pp |  |
| $\mathrm{C}_{\text {max }}$ for PM | 1.4 (m |  |
| $G_{\text {NOX }}$ | Weighted mean of curve L \& | ( $\mathrm{m}^{3} / \mathrm{km} \cdot \mathrm{veh}$ ) |
| $G_{\text {PM }}$ | Weighted mean of curve L \& | (g/km $\cdot \mathrm{veh}$ ) |
| $L_{a i} L_{\text {ao }}$ | $25.8\left(=3 d_{t}\right)$ |  |
| $R_{\text {et }}$ | $1.3 \times 10^{6} \sim 5.1 \times 10^{6}$ |  |
| $v$ | $40 \sim 90$ |  |
| $\alpha$ | 1.0 ( $l_{V} / d_{v} \geqq 16.75$ ) Eq.(8) |  |
| $\gamma_{L}$ | $0,10,20,30,40,50,75,100$ |  |

The $\mathrm{NO}_{\mathrm{x}}$ emission per unit distance, $G_{\mathrm{Nox}}$, and the corresponding length limit $L_{\text {lim }}$ are plotted in Fig. 6 (a) for given vehicle speed $v$ and large vehicle ratio $\gamma_{L}$. The corresponding result for PM is given in (b). For a typical traffic condition of vehicle speed $\boldsymbol{v}=80 \mathrm{~km} / \mathrm{h}$ and large vehicle ratio $\gamma_{L}=20 \%$, the length limit can be expressed as:

$$
L_{\mathrm{lim}}=640 \mathrm{~m}\left[G_{\mathrm{NOx}}=2.08 \times 10^{-3} \mathrm{~m}^{3} /(\mathrm{km} \cdot \mathrm{veh})\right], L_{\mathrm{lim}}=435 \mathrm{~m}\left[G_{\mathrm{PM}}=0.394 \mathrm{~g} /(\mathrm{km} \cdot \mathrm{veh})\right]
$$

The result obtained justifies the common understanding among tunnel specialists that road tunnels up to several hundred meters need no artificial ventilation, even for two-way traffic tunnels.

## 4 CONCLUSIONS

A concept of diffusive ventilation is introduced for the design and planning of tunnel ventilation. To identify the intensity of diffusive ventilation, the tunnel space is assumed as a one-dimensional turbulence field and longitudinal diffusion coefficient LDC is adopted as the index of diffusive ventilation.

Series of experiments were conducted to evaluate LDC using model and actual tunnels. Effort was made to form a simplified and comprehensive correlation equation for obtaining LDCs. The equation proposed in the present paper enables to correlate LDCs by using only the equivalent resistance area of vehicles, traffic flow rate and vehicle Reynolds Number.

As an example of application of LDC, length limit of naturally ventilated tunnels for two-way traffic is estimated under the condition that the concentration of a specific pollutant in the tunnel does not exceed the admissible limit. Limiting length is calculated for two pollutants, $\mathrm{NO}_{\mathrm{x}}$ and PM , and the result justified the common understanding among tunnel specialists that road tunnels up to several hundred meters need no artificial ventilation, even for two-way traffic tunnels.

The application of LDC for the planning and design of tunnel ventilation has been expanding in such situations that diffusive transfer becomes comparable with convective transfer, that is, through-flow velocity is very low or concentration gradient is very large.

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## APPENDIX - Measurement and analysis of LDCs in an actual tunnel by bursting a tracer gas filled balloon

Measurement and analysis of LCDs in an actual tunnel, including results, is outlined in the following.

## 1 MEASUREMENT METHOD

The experiment on longitudinal diffusion was carried out with actual vehicles running in the Metropolitan Expressway Kawasaki Fairway Tunnel (west-bound, overall length 1,954 m, cross-sectional area $A_{r}=87.2 \mathrm{~m}^{2}$, equivalent diameter of tunnel $d_{r}=8.6 \mathrm{~m}$, one-way, threelanes) (Ref.4). Appendix Fig. 1 shows a conceptual diagram outlining the measurement. A given amount of tracer gas ( $\mathrm{SF}_{6}$ ) was released instantly into the tunnel at Point A (by bursting a gas-filled balloon). The tracer gas drifted in the longitudinal direction along the air flow, dispersed by passing vehicles and spreading throughout the tunnel, the cross-section eventually reaching a uniform concentration distribution. The sampling point of the tunnel air, Point B , is located about 1 km downstream of the tracer source, a sufficient distance to develop into onedimensional diffusion field. The tracer gas concentration was measured continuously by a gas chromatograph and pen recorder. The longitudinal through-flow velocity in tunnel $U_{r}$ was obtained from the travel time necessary to move from A to B. The amount of traffic and vehicle speed were also measured for 5 minutes during the experiment using a video camera and a cassette video recorder. The mass of diffused tracer gas was evaluated by subtracting the tare weight from the balloon weight and adding the buoyancy force.


## Appendix Fig. 1 Outline of measurement

Appendix Fig. 2 shows an example of actually measured concentration distribution at the observation point $B$.


## Appendix Fig. 2 Example of recording concentration distribution

## 2 ANALYSIS METHOD

The instantly released tracer gas first diffuses into the surrounding air in a three-dimensional manner. Then, along the progression of cross-sectional uniformity, it starts to diffuse in the longitudinal direction and turns into a one-dimensional diffusion. If we observe the diffusion process moving along with the longitudinal through-flow velocity in tunnel $U_{r}$, it can be observed that the highest concentration point stays always at the center and concentration distribution becomes gradually flat as diffusion proceeds. This diffusion process must follow Fick's law describing one-dimensional diffusion in a stationary bulk.

Assuming the distance from the highest concentration point as $\xi$ and the time following the start of diffusion as $t$, the concentration distribution can be expressed using Fick's law as the following

$$
C(\xi, t)=\frac{Q}{2 A_{r} \sqrt{\pi D t}} \exp \left(-\frac{\xi^{2}}{4 D t}\right) \quad \text { Appendix equation (1) }
$$

where, $Q$ is the amount of released tracer gas.

The distance $\xi$ from the highest concentration point to the observation point $x=x_{0}$ is $x_{0}$ at the beginning $t=0$, and decreases as time proceeds. The relation is given by

$$
\xi=x_{0}-U_{r} t
$$

Substituting the above relation to Fick's law, the concentration at the observation point is obtained by:

$$
C\left(x_{0}, t\right)=\frac{Q}{2 A_{r} \sqrt{\pi D t}} \exp \left(-\frac{\left(x_{0}-U_{r} t\right)^{2}}{4 D t}\right) \quad \text { Appendix equation (2) }
$$

The maximum concentration $C_{\text {max }}$ emerges at $t_{0}=x_{0} / U_{r}$ and therefore

$$
C_{\max }=\frac{Q}{2 A_{r} \sqrt{\pi D t_{0}}}
$$

Appendix equation (3)
In the above discussion we used the time $t$ starting from the moment of the burst. This is equivalent to assume that the whole diffusion process follows Fick's law for one-dimensional diffusion. This is, of course, not the case because three-dimensional diffusion takes place till the tracer gas fills the tunnel section. The time $t$ in Appendix equation (2) must be counted from the virtual burst time at which one-dimensional diffusion starts with infinitely large concentration. This virtual start time is impossible to estimate reasonably. In conclusion, we decided to neglect the gap between real and virtual burst and to accept the error that might occur as the consequence.


Appendix Fig. 3 Concentration distribution
Appendix Fig. 3 shows an example of comparison between measured and calculated change of concentration at the observation point. The black dots in the figure represent measured data, while the solid curve is calculated by Appendix equation (2) whose $D$ value is determined by the best-fitting. The method to find $D$ value that fits the measured data the most is described in the following steps.

1. By substituting measured maximum concentration $C_{\text {max }}$, volume of tracer gas $Q$, traveling time $t_{0}$ and cross sectional area $A_{r}$ to Appendix equation (3), $D$ can be determined as the first assumed value.
2. Put assumed $D$ to Appendix equation (2) and calculate concentration distribution around its peak.
3. Compare the measured and calculated distribution around the peak as in Appendix Fig. 3 and evaluate the coefficient of correlation (residual sum of squares and sum of squared deviation) between measurement and calculation.
4. Increase or decrease the assumed $D$ stepwise and repeat the step 2 and 3 till the maximum coefficient of correlation is found at a certain $D$ value.
5. Adopt this $D$ as the final value that fits the measured data the best.

## 3 RESULTS

### 3.1 Traffic rate and tunnel through-flow

Appendix Table 1 shows a list of measured values on the traffic rate and tunnel through-flow. Mean values of traffic rate $N$, vehicle velocity $v$ and large-vehicle ratio $\gamma_{L}$, measured in fiveminute duration are shown. The longitudinal through-flow velocity in tunnel $U_{r}$ was obtained from the travel time and distance of tracer gas. Also shown in the table is the value of equivalent resistance area of vehicle $A_{m}$ which was calculated using actually measured traffic conditions and using an equation proposed by Ohashi et al. (Ref.12).

## Appendix Table 1 Traffic conditions and tunnel through-flow

|  | $N$ <br> veh/s | $v$ <br> $\mathrm{~m} / \mathrm{s}$ | $\gamma_{L}$ <br> $\%$ | $d_{v}$ <br> m | $A_{m}$ <br> $\mathrm{~m}^{2}$ | $U_{r}$ <br> $\mathrm{~m} / \mathrm{s}$ |
| :--- | :--- | :--- | :---: | :--- | :--- | :--- |
| Run1 | 0.383 | 25.97 | 57.4 | 2.47 | 3.68 | 7.3 |
| Run2 | 0.360 | 26.94 | 50.7 | 2.38 | 3.35 | 7.4 |
| Run3 | 0.350 | 25.97 | 51.4 | 2.39 | 3.38 | 6.9 |
| Run4 | 0.323 | 28.19 | 51.5 | 2.39 | 3.38 | 7.4 |
| Run15 | 0.403 | 25.42 | 45.5 | 2.31 | 3.08 | 6.8 |
| Run16 | 0.377 | 27.36 | 37.2 | 2.20 | 2.67 | 6.9 |
| Run17 | 0.383 | 28.61 | 35.7 | 2.18 | 2.59 | 6.8 |
| Run18 | 0.330 | 27.36 | 29.3 | 2.18 | 2.27 | 6.8 |
| Run19 | 0.357 | 26.53 | 43.0 | 2.28 | 2.96 | 6.7 |
| Run20 | 0.383 | 25.69 | 34.8 | 2.17 | 2.55 | 6.1 |
| Run21 | 0.310 | 27.64 | 30.1 | 2.11 | 2.31 | 5.8 |

### 3.2 Estimation of the longitudinal diffusion coefficient

Appendix Table 2 shows the vehicle Reynolds Number $R_{e t}$ and estimated longitudinal diffusion coefficient $D$ for each run.

The LDCs in this table are plotted in Fig. 3 of the present paper as the data measured in a real tunnel.

## Appendix Table 2 Determined values of vehicle Reynolds Number $\boldsymbol{R}_{\boldsymbol{e t}}$ and longitudinal diffusion coefficient $D$

|  | $R_{e t}$ | $D \mathrm{~m}^{2} / \mathrm{s}$ |
| :--- | :--- | :--- |
| Run1 | $4.3 \times 10^{6}$ | 97.0 |
| Run2 | $4.3 \times 10^{6}$ | 85.8 |
| Run3 | $4.1 \times 10^{6}$ | 87.7 |
| Run4 | $4.5 \times 10^{6}$ | 72.3 |
| Run15 | $3.9 \times 10^{6}$ | 79.6 |
| Run16 | $4.0 \times 10^{6}$ | 92.5 |
| Run17 | $4.2 \times 10^{6}$ | 86.0 |
| Run18 | $3.8 \times 10^{6}$ | 64.9 |
| Run19 | $4.0 \times 10^{6}$ | 97.1 |
| Run20 | $3.7 \times 10^{6}$ | 75.8 |
| Run21 | $3.9 \times 10^{6}$ | 68.3 |

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[^0]:    Note: Small vehicles: all passenger cars, micro-busses of less than 30 passengers and small trucks width less than 1.7 m . Large vehicles: other busses, trucks, trailers and special vehicles.

